9^0

1. $a = 12t^2 - 4$

$v = \frac{12t^3}{3} - 4t + c$

Initially at rest $v(0) = 0$

$0 = 4(0)^3 - 4(0) + c$

$c = 0$

$\boxed{v = 4t^3 - 4t}$

$0 = 4t(t^2 - 1)$

$= 4t(t+1)(t-1)$

$t = 0$ $\cancel{t = -1}$ $t = 1$

b. $x(t) = \frac{4t^4}{4} - \frac{4t^2}{2} + c$

$= t^4 - 2t^2 + c$ $\quad x(1) = 3$

$3 = 1^2 - 2(1)^2 + c$

$4 = c$

$x(t) = t^4 - 2t^2 + 4$

	-1	0	1
$4t$	—	0	
$t+1$	-0		
$t-1$	—		0
		$0\ -$	$0\ +$

c. Total distance

0 to 1 $\quad x(0) = 0^4 - 2(0)^2 + 4 = 4$

1 to 2 $\quad x(1) = 1^4 - 2(1)^2 + 4 = 1 - 2 + 4 = 3$

$x(2) = 2^4 - 2(2)^2 + 4 = 16 - 8 + 4 = 12$

$4 - 3 = 1$

$12 - 3 = 9$

$\overline{\quad 10}$

d. AVG Velocity $= \frac{d}{t} = \frac{10}{2} = 5$

$\frac{200 \text{ miles}}{4 \text{ hrs}} = 50 \text{ mph}$

90/2. $y = \ln\left(\frac{x}{x-1}\right)$

$x \neq 1$

$\frac{x}{x-1} > 0$

		0		1	
x		0			
$x-1$				0	
	+	0	−	0	+

$x > 1$

$x < 0$

b. $\frac{1}{\frac{x}{x-1}} \left[\frac{(x-1)(1) - x(1)}{(x-1)^2}\right]$ evaluate at $x = -1$

$= \frac{1}{\frac{-1}{-1-1}} \left[\frac{(-1-1)1 - (-1)}{(-1-1)^2}\right]$

$= \frac{1}{\frac{-1}{-2}} \left[\frac{-2+1}{4}\right] = 2\left[\frac{-1}{4}\right] = -\frac{1}{2}$

c. $y = \ln\left(\frac{x}{x-1}\right)$

$x = \ln \frac{y}{y-1}$

$e^x = e^{\ln \frac{y}{y-1}}$

$e^x = \frac{y}{y-1}$

$(y-1)e^x = y$

$ye^x - e^x = y$

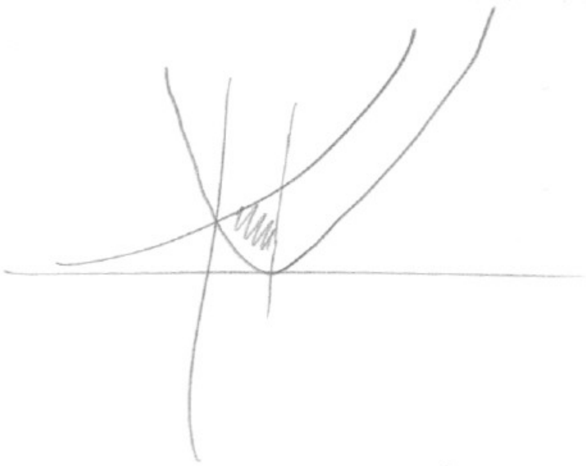
$ye^x - y = e^x$

$y(e^x - 1) = e^x$

$y = \frac{e^x}{e^x - 1}$ or $y = \frac{1}{e^x - 1} + 1$

$f^{-1}(x) = \frac{e^x}{e^x - 1}$

$\frac{90}{3}$



a. $\int_0^1 \left(e^x - (x-1)^2\right) dx$

$= 1.385 \quad = e - \frac{4}{3}$

b. $\pi \int_0^1 \left[(e^x)^2 - ((x-1)^2)^2\right] dx$

$= 2.995\pi \quad = \frac{1}{2}\pi e^2 - \frac{7}{10}\pi$

c) $2\pi \int_0^1 x\left(e^x - (x-1)^2\right)$

90/4. $\frac{dr}{dt} = 0.04$

$V = \frac{4}{3}\pi r^3$

a. $\frac{dv}{dt} = 4\pi r^2 \frac{dr}{dt}$

$= 4\pi(10)^2(0.4)$

$= 4\pi 100(.04)$

$= 16\pi$

b. $V = 36\pi$

$V = \frac{4}{3}\pi r^3$

$36\pi = \frac{4}{3}\pi r^3$

$27 = r^3$

$r = 3$

$A = \pi r^2$

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

$= 2\pi(3)(.04)$

$= \frac{6\pi}{25}$

c. $\frac{dv}{dt} = \frac{dr}{dt}$ $r = ?$

$\frac{dv}{dt} = .04$

$V = \frac{4}{3}\pi r^3$

$\frac{dv}{dt} = 4\pi r^2 \frac{dr}{dt}$ not necessary to substitute the value

$(.04) = 4\pi r^2(.04)$

$1 = 4\pi r^2$

$\frac{1}{4\pi} = r^2$

$r = \sqrt{\frac{1}{4\pi}}$ cm

OR

$V = \frac{4}{3}\pi r^3$

$\frac{dv}{dt} = 4\pi r^2 \frac{dr}{dt}$ $\frac{dv}{dt} = \frac{dr}{dt}$

$\frac{dr}{dt} = 4\pi r^2 \frac{dr}{dt}$

$1 = 4\pi r^2$

$\frac{1}{4\pi} = r^2$

$\sqrt{\frac{1}{4\pi}} = r$

90/

5. $f(x) = \sin^2 x - \sin x$

a) $0 = \sin^2 x - \sin x$

$= \sin x(\sin x - 1)$

$\sin x = 0 \qquad \sin x - 1 = 0$

$\qquad\qquad\qquad \sin x = 1$

$0, \pi \qquad\qquad \frac{\pi}{2}$

b) $y' = 2\sin x \cos x - \cos x$

$= \cos x(2\sin x - 1)$

$\cos x = 0 \qquad 2\sin x - 1 = 0$

$\qquad\qquad\qquad 2\sin x = 1$

$\qquad\qquad\qquad \sin x = \frac{1}{2}$

$\frac{\pi}{2}, \frac{3\pi}{2} \qquad\qquad \frac{\pi}{6}, \frac{5\pi}{6}$

	$\frac{\pi}{6}$	$\frac{\pi}{2}$	$\frac{5\pi}{6}$	$\frac{3\pi}{2}$
$\cos x$	+ + +	0 - - -	- - - -	0
$2\sin x - 1$	- - 0 +	+ + +	0 - - -	
	- 0 +	0 -	0 +	0

Increas $\left(\frac{\pi}{6}, \frac{\pi}{2}\right) \quad \left(\frac{5\pi}{6}, \frac{3\pi}{2}\right)$

c. Evaluate

$\frac{\pi}{6}$

$\frac{\pi}{2}$

$\frac{5\pi}{6}$

$\frac{3\pi}{2}$

MAX $\left(\frac{\pi}{2}, 0\right)$

MIN $\left(\frac{\pi}{6}, -\frac{1}{4}\right)$

MAX $\left(\frac{3\pi}{2}, 2\right)$

$\left(\frac{5\pi}{6}, -\frac{1}{4}\right)$

9% 6. $f(x) = \dfrac{ax+b}{x^2-c}$

i) symetric about y axis

$\therefore$ Even

ii) $\lim\limits_{x \to 2^+} f(x) = \infty$

iii $f'(1) = -2$

(i) $\dfrac{ax+b}{x^2-c} = \dfrac{a(-x)+b}{(-x)^2-c}$ Even

$\therefore$ a must be zero

eqn becomes

(ii) $\dfrac{b}{x^2-c}$

if $\lim\limits_{x \to 2} f(x) = \infty$

$x^2 - c = 0$

$2^2 - c = 0$

$\therefore c = 4$

eqn becomes

$\dfrac{b}{x^2-4}$

(iii) $\dfrac{(x^2-4)(0) - b(2x)}{(x^2-4)^2}$ $f'(1) = -2$

$\dfrac{-b(2(1)}{(1^2-4)^2} = \dfrac{-2b}{9}$

$\dfrac{-2b}{9} = -2$

$-2b = -18$

$b = 9$

1990

6 cont

$$f(x) = \frac{9}{x^2-4}$$

~~Vertical~~ Horiz Asym

$$\lim_{x \to \infty} \frac{9}{x^2-4} = 0$$

$$\boxed{y=0}$$

Vertical $x^2-4=0$

$$\boxed{x=\pm 2}$$

x	$\frac{9}{x^2-4}$
3	$\frac{9}{3^2-4} = \frac{9}{5}$
0	$\frac{9}{0^2-4} = -\frac{9}{4}$
-3	$\frac{9}{(-3)^2-4} = \frac{9}{5}$